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# RESEARCH MEMORANDUM

## PRELIMINARY ANALYSIS OF PROBLEM OF DETERMINING EXPERIMENTAL PERFORMANCE OF AIR-COOLED TURBINE

## I - METHODS FOR DETERMINING HEAT-TRANSFER CHARACTERISTICS


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# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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RESEARCH MEMORANDUM

# PRELIMINARY ANALYSIS OF PROBLEM OF DETERMINING EXPERIMENTAL PERFORMANCE OF AIR-COOLED TURBINE

## I - METHODS FOR DETERMINING HEAT-TRANSFER CHARACTERISTICS

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## SUMMARY

Little material has been published on the methods of investigating air-cooled turbines and analyzing the data obtained to evaluate the turbine performance, which includes the heat-transfer and cooling-air-flow characteristics in addition to the performance characteristics normally obtained in uncooled turbines. Some analysis and study has been made by the NACA Lewis laboratory in connection with investigations that are to be made on air-cooled turbines. Methods that are required to determine the characteristics mentioned are presented. The methods have a form in which dependent parameters are expressed as functions of independent parameters, the functions generally being unknown. Methods of experimenting to determine these functions are suggested. Although the forms of the formulas are, for the most part, unchecked experimentally, it is believed that the information will be useful in the turbine-cooling field.

The methods that are expected to permit determination of the heat-transfer characteristics of an air-cooled turbine from specific investigations that must be conducted are presented herein. In some cases discussions that lead to the suggested form of the formulas are given in order that readers unfamiliar with the subject may obtain a knowledge of the general background of heat transfer.

## INTRODUCTION

Little material is available at present on the methods of investigating air-cooled turbines and analyzing the data obtained to evaluate their performance. The performance of turbines formerly included such factors as efficiency and power but did not include characteristics such as heat transfer and cooling-air flow because

the turbines were uncooled. Some study and analysis has been made at the NACA Lewis laboratory on the problem of investigating and evaluating the performance of air-cooled turbines. Because manufacturers are applying cooling to experimental turbines, the results of the studies at the Lewis laboratory, although lacking experimental verification, are presented in a series of reports.

The reports will present suggested formulas that are required to determine air-cooled turbine performance. The formulas will be set up so that certain dependent parameters are functions of independent parameters, the functions in general being unknown. Methods of investigation to determine these functions will be suggested. The suggested methods for determining the heat-transfer characteristics, the cooling-air flow characteristics, and the power and the efficiency of air-cooled turbines will be presented. These reports will be concerned only with the turbine. The problem of power loss in the engine due to bleedoff of cooling air from the compressor when high pressure is required and other engine problems are not included in the evaluation of turbine performance.

Methods based upon present knowledge that will permit determination of the heat-transfer characteristics of a turbine from specific investigations, which must be conducted, are suggested herein. In some cases discussions are given in order that those unfamiliar with the subject may obtain some knowledge of the general background of heat transfer. From the heat-transfer characteristics, factors such as the maximum permissible gas temperature, the amount of heat added to the coolant, and the cooling-air requirements, which are of primary importance in determining the power of the turbine or the thrust of the engine, can be obtained for conditions other than those investigated.

## METHODS OF EVALUATING HEAT-TRANSFER CHARACTERISTICS

### Factors Required

It is important to be able to determine the gas temperature at which a cooled turbine operates safely for a given coolant flow and coolant temperature (or, the coolant conditions that are required to operate at a given gas temperature without exceeding blade-stress limitations). From such a determination, the net performance of the turbine or engine can be found by use of suitable formulas.

In order to determine the safe operating conditions, the allowable-blade-temperature-distribution curve and the actual-blade-temperature-distribution curve must be found. The actual-blade-temperature-distribution equation involves heat-transfer coefficients and effective fluid temperatures for which formulas must be available.

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As heat is absorbed by the coolant, the performance of the turbine varies. Consequently, a parameter involving the heat to the coolant enters into the performance determination and formulas for the calculation of this parameter must be set up using test data. Discussions of the details of the heat-transfer formulas and secondary formulas, which are required to obtain the heat-transfer formulas, are given. The discussions are concerned with the rotor, except where otherwise noted. The last part of this section is concerned with the cooled nozzle for which the methods of evaluating the rotor heat-transfer characteristics can be applied with only small changes.

### Allowable Radial Blade Temperature Distribution

The manner of determining the coolant flow required for safe operation of the turbine at given effective gas and coolant temperatures is shown in figure 1. For each turbine speed an allowable-radial-temperature-distribution curve exists. Also for a given effective gas temperature $T_{g,e}$ and effective coolant temperature $T_{a,e}$, the coolant flow determines the actual radial temperature distribution, as illustrated in figure 1. A particular coolant flow gives a curve such as A (fig. 1), which will contact the allowable temperature curve at some point such as C. This point then represents the lowest allowable coolant flow for the given operating conditions. A lower flow would cause curve A to move upward and exceed the allowable temperature over a portion of the blade, causing blade failure.

The allowable blade temperature depends on the stress characteristics of the blade. A brief discussion of this subject is given in reference 1. The conclusion of the discussion is that, for the present, the allowable blade temperature distribution must be determined from the radial centrifugal stress distribution in the blade and from stress-rupture data. The method of determining this allowable blade temperature distribution, which is quite simple, is reviewed herein for convenience.

The centrifugal-stress distribution is determined by calculating the stress, increment by increment, from the tip to the root of blade. (See fig. 2(a).) The centrifugal force of an increment is

$$\text{centrifugal force} = \rho_B A_{B,av}\, dr\, \omega^2 r \qquad (1)$$

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The area $A_{B,av}$ is the blade cross-sectional area at the center of the increment in question. (All symbols are defined in appendix A.) The blade stress at any section is the sum of the centrifugal forces acting on all the increments from the section to the tip divided by the area of the section, or

$$S = \frac{B + \rho_B A_{B,av} \, dr \, \omega^2 r}{144 A_B} \tag{2}$$

where $A_{B,av}$, $A_B$, $r$, and $dr$ correspond to the increment under consideration and $B$ is the sum of the centrifugal forces on preceding increments. The area $A_B$ is that of the base of the increment. (See fig. 2(b).) The stress at any portion of the blade can be determined by this step-by-step method. In some cases where the areas of the blade sections can be represented as a function of $r$, the stress distribution can be obtained by integration.

When the centrifugal-stress distribution is known, the allowable radial temperature distribution can be determined from stress-rupture data. The 10,000-hour life stress-rupture curve has usually been used in order to obtain a factor of safety because of the vibratory and thermal stresses that are neglected.

## Actual Blade Temperature Distribution

The actual blade temperature distribution formula can be considerably simplified if the following assumptions are made:

(a) Radiation effects are negligible.

(b) Wall thickness is negligible insofar as heat transfer is concerned; that is, no temperature drop occurs through the wall.

(c) Conduction of heat in the radial and peripheral directions in the blade wall is negligible.

(d) Blade mean cross section applies to the entire blade when average-value calculations are made.

(e) Local convection heat-transfer coefficients on the inside and outside of the blade are constant spanwise and peripherally at given operating conditions.

(f) Local effective combustion-gas temperature (temperature effecting heat transfer on outside of blade) is constant over
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With these assumptions the equation for the blade temperature at any point $x$ along the span from heat-transfer considerations is

$$\frac{T_{g,e} - T_{B,x}}{T_{g,e} - T_{a,e,h}} = \left(\frac{1}{\lambda+1} + X - Y\right)e^{-Z} - \frac{\omega^2 w_a b}{JgH_o l_o \left(T_{g,e} - T_{a,e,h}\right)} \frac{x}{b} - X + Y \tag{3}$$

Equation (3) can be derived from equations given in appendix B of reference 2.

Radiation between the blades and other surfaces and radial heat conduction has little effect on the temperature distribution except near the blade root (reference 2). This fact, however, does not appreciably affect the determination of the required coolant flow or the allowable gas temperature. Gaseous radiation may possibly become a factor to consider in some special cases such as very high gas temperatures and gases containing certain constituents. In general, however, gaseous radiation can be neglected. Assumption (b) should apply well for the thin-walled blades used in air-cooled turbines. The neglect of peripheral heat conduction, assumption (c), is warranted for the greatest part of the blade on the basis of unpublished investigations already made on cascades. The error involved through use of assumptions (d), (e), and (f) must await comparison of temperatures calculated from equation (3) and experimental values. On the basis of some study, it is believed that the use of these assumptions will result in small error in the blade temperature distribution.

The use of assumption (b) and neglect of heat conduction in a peripheral direction reduces the heat transfer to a one-dimensional problem, that of radial heat transfer, and equation (3) is consequently the equation for the radial temperature distribution. For a given set of combustion-gas and cooling-air conditions, all terms in equation (3) are constants except $T_{B,x}$ and $x$. Thus the equation reduces to one of simple form for use in the analysis of a turbine. In some cases the $X$ and $Y$ terms can also be neglected resulting in further simplification.

Before applying equation (3) to determine the blade temperatures of a given turbine, equations for $H_o$, $H_i$, $T_{g,e}$, and $T_{a,e}$ must be determined from data obtained in experiments on the turbine. These equations are now discussed.

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## Outside Convection Heat-Transfer Coefficient

Method of calculation. - The heat addition to the cooling air as it passes through the blades, the average effective gas temperatures, and the average blade temperature must first be determined from experimental data. With these data and the blade dimensions, the outside convection heat-transfer coefficient $H_o$ is calculated. The following equation is used:

$$H_o = \frac{Q_a}{l_o b \left(T_{g,e,av} - T_{B,av}\right)} \tag{4}$$

This equation again assumes that all radiation is negligible. When radiation is not negligible, the discussion in the previous section is applicable. Discussions of methods for determining $Q_a$, $T_{g,e,av}$, and $T_{B,av}$ from experimental measurements are discussed in the following paragraphs.

Heat gained by cooling air $Q_a$. - From the general energy equation, it can be shown that the heat gained by the cooling air in the blade cooling-air passage for a short distance $dr$ can be expressed by the following equation:

$$dq_a - d\Omega = c_{p,a}\, dT_a + d\left(\frac{W_a^2}{2Jg}\right) \tag{5}$$

where

$dq_a$ differential quantity of heat gained by cooling air, (Btu/lb cooling air)

$d\Omega$ work added to cooling air by blade rotation, (Btu/lb cooling air)

$dT_a$ rise in static temperature of cooling air, (°F)

It can be shown that the right-hand side of equation (5) equals $c_{p,a}\, dT''_a$ where $dT''_a$ is the rise in total temperature of the cooling air relative to the blade. The work is

$$d\Omega = -\frac{\omega^2 r\, dr}{Jg} \tag{6}$$

Thus equation (5) reduces to

$$dq_a = c_{p,a}\, dT''_a - \frac{\omega^2 r\, dr}{Jg} \tag{7}$$

By integrating from the roots to the tips of the blades and by assuming $c_{p,a}$ to be a constant but determined at the mean of the static temperatures at the root and the tips, equation (7) becomes

$$q_a = c_{p,a,av}\left(T''_{a,T} - T''_{a,h}\right) - \frac{\omega^2}{2Jg}\left(r_T^{\,2} - r_h^{\,2}\right) \tag{8}$$

The total heat to cooling air is then

$$Q_a = w_a q_a \tag{9}$$

Because of circulatory effects of the fluid in the passages, the angular velocity of the cooling air, which is the term that should be used in equation (8), is a little smaller than the angular velocity of the wheel $\omega$, which is used in equation (8). Consequently, it is more accurate to write equation (8) as

$$q_a = c_{p,a,av}\left(T''_{a,T} - T''_{a,h}\right) - f_s\,\frac{\omega^2}{2Jg}\left(r_T^{\,2} - r_h^{\,2}\right) \tag{10}$$

where $f_s$ is the factor (equivalent to the slip factor of a centrifugal supercharger) to correct for the circulatory effects of the cooling air. The value of this factor can be determined by motoring the turbine wheel at several speeds and, with no gas passing across the turbine blades, allow various cooling-air quantities to pass through the blades. Thus $q_a$ will be 0 and $f_s$ can be calculated from equation (10) from the measured air temperatures and set up as a function of $w_a$ and $\omega$.

The total temperatures of the air relative to the blade at the root and the tip are determined by means of the usual formulas involving the recovery factor of the thermocouples, obtained from calibration tests, measured temperatures, and measured values of static and total pressures in the cooling-air passages. The static- and total-pressure measuring devices are placed as close to the thermocouples as possible. A more detailed discussion of these measurements is given later because obtaining measurements in rotating passages involves some special problems.

It is important to be able to determine $Q_a$ for the turbine from the established formulas in order that the performance of an engine in which the turbine is to be placed can be calculated for any flight and engine conditions. Equation (10) can be used if the cooling-air total temperatures can be calculated from test-verified formulas. The methods of obtaining $T''_a$ for any condition so that $Q_a$ can be calculated are discussed in reference 3.

Effective gas temperature $T_{g,e}$. - The temperature effecting heat transfer from the gas is equal to the temperature the blades would assume if they were thermally insulated (no heating or cooling) under the same fluid conditions for which the heat-transfer coefficient is being determined. The effective gas temperature thus is the adiabatic blade-surface temperature. The reason for the use of such a temperature to determine convection heat-transfer coefficients is discussed in reference 1.

The adiabatic temperature of a body such as a turbine blade, like that for a thermocouple, can be determined, as brought out in reference 1, from the total and static temperatures of the fluid flowing around or through the body and from a recovery factor. Thus it is possible to conduct investigations in which no cooling air is used and to determine the average total and static temperatures about the blades as well as the blade temperatures. The recovery factor for the blades can then be found from the equation

$$\Lambda_g = 1 - \frac{T''_{g,av} - T_{g,e}}{T''_{g,av} - T_{g,av}} \tag{11}$$

where $T_{g,e}$ is equal to the average of the measured blade temperatures $T_{B,av}$ under adiabatic conditions.

A summary of the studies of many investigators on various bodies, including that of reference 4 on turbine blades, to determine the parameters affecting the effective gas temperature is given in reference 1. In general a formula of the following form is applicable:

$$\frac{\Lambda_g}{(Pr_g)^n} = f^I(M_{g,3}) \tag{12}$$

where $f^I$ denotes an unknown function and $n$ an exponent of the Prandtl number. For the case of laminar boundary layer on a flat plate the exponent $n$ has been shown to be equal to 1/2.

Tests must therefore be conducted on the turbine over a range of Prandtl and Mach numbers at gas temperatures that are safe when no cooling air is passed through the blades to establish the function of the formula given by equation (12).

It is possible, however, that adiabatic conditions are not obtainable at temperatures much different from those surrounding the experimental setup. Also, over the probable range of temperatures covered in this type of investigation, the Prandtl number will not vary more than 5 percent and this number raised to a power less than 1.0 will vary even less. The accuracy of the data may therefore not warrant an investigation varying the Prandtl number. The recovery factor $\Lambda_g$ is calculated using equation (11). After establishing equation (12), effective gas temperatures for use in equation (4) for the conditions of high gas temperature used when cooling air is passed through the blade, can be calculated using equation (11), the recovery factor obtained through use of the established equation (12), and measured fluid conditions.

For the case of establishing equation (12) from low-temperature experiments with no cooling air and the case of calculation of $T_{g,e}$ for high-temperature experiments with cooling air to determine $H_o$, methods for determining the average relative total gas temperature $T''_{g,av}$, the average static gas temperature $T_{g,av}$, the Prandtl number $Pr_g$, and the average Mach number $M_{g,3}$ from measurements made during the investigations must be known. Also the method of determining the average blade temperature $T_{B,av}$ in establishing equation (12) must be known. These methods are described in the following sections.

It is recommended that experiments with variable Mach number be conducted at several constant gas Reynolds numbers to verify the neglect of the Reynolds number in equation (12). (See reference 4.) The Mach number variation can be obtained by varying the fluid flow rate and turbine speed. Evidence exists that the formula for $\Lambda_g$ should be based on the Mach number at the rotor-blade outlet $M_{g,3}$ (references 4 and 5). The reason for the variation of the Mach number basis of $\Lambda_g$ is thought to be due to the boundary-layer flow conditions that exist in each case. Until more data are available, it is recommended that the Mach number that correlates the data be used.

Because the differences between the fluid temperatures and blade temperatures when determining $A_g$ are very small, the blade temperatures should be measured differentially with respect to some one fluid temperature if any degree of accuracy is to be obtained.

Average blade temperature $T_{B,av}$. - It is possible to connect the various thermocouples used to measure the blade-wall temperatures if they are insulated from each other in a parallel circuit so that the reading of the measuring instrument used will be an average for the given test conditions imposed on the turbine. A circuit of this type, however, might cause mechanical difficulties when used on a rotating wheel and the average may not be an integrated average.

It is suggested that individual thermocouples be placed in peripheral bands at various stations along the blade span. The thermocouple readings can then be integrated with respect to the distance around the blade for each peripheral band. Then if the integrated mean values of temperature for the peripheral bands are integrated across the blade span, the average blade temperature $T_{B,av}$ is obtained.

Total temperature of cooling air $T''_a$. - The observed temperatures of the cooling air at the blade root and tip can be obtained by means of a thermocouple placed at each position in the blade cooling-air passage. It is assumed that any thermocouple placed at each station will be so located that a good average of the air temperature across the passage is obtained by the one reading. Because the thermocouples are attached to the blades, the total temperatures calculated from the observed readings will be values relative to the blade, which are the values required.

It is quite likely that the thermocouple readings will be in error if improperly shielded and insulated because of radiation from the hot blade walls and conduction from the blade walls along the thermocouple support to the junction. Much care and consideration should therefore be given to the installation of the thermocouples.

As previously mentioned, the total temperatures $T''_{a,h}$ and $T''_{a,T}$ required in equation (10) are calculated from the observed readings, the recovery factor of the thermocouple, and the pressures in the passage. The recovery factor is also a function of

the Mach number of the air so that the pressures at each station are also required to determine the Mach numbers. These pressures
1237 are discussed in the following section.

Cooling-air pressures in blade passage. - The total pressures $p''_a$ and static pressures $p_a$ of the cooling air at the blade root and tip can be obtained by means of pressure tubes attached to the blade and static-pressure taps in the blade wall. It is desirable that static-pressure tubes be located in the cooling-air passage in addition to use of taps in the wall in order to get a true average pressure. In most cases, it will probably be impossible to put more than one or two instruments in any one passage. If more than one instrument is required to obtain an average pressure, these instruments will have to be apportioned among several blades.

The pressures in the rotating-blade coolant passages are transmitted to a point near the center of the turbine through tubing and then through a pressure pickup to a stationary manometer board. The centrifugal effects in the interconnecting tubing must be taken into account when evaluating the pressures in the passage from the manometer-board readings. The equation for using the manometer-board readings to obtain the correct pressures in the coolant passages is derived in appendix B. For pressures, either total or static, obtained with pressure tubes located at the blade root, the following equation is applicable:

$$\text{Rotating pressure} = (\text{manometer differential pressure} \pm \text{pressure of room in which manometer is placed})\, e^{\frac{\omega^2 r_h^2}{2gR_a T_{a,t}}} \tag{13}$$

The temperature $T_{a,t}$ is the temperature of the fluid in the connecting pressure tube, which is discussed in appendix B. For pressures at the blade tip, $r_T$ is substituted for $r_h$ in equation (13).

Average static gas temperature $T_{g,av}$. - The ideal method of obtaining an accurate average static temperature would entail measurements through the gas passage between the blades and then integration of the resulting values with respect to the blade-passage length. This method, however, requires a large turbine and elaborate instrumentation. A more practical indication of the average static temperature can be obtained from the expression

$$T_{g,av} = \frac{T_{g,2} + T_{g,3}}{2} \tag{14}$$

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where

$T_{g,2}$ static temperature of gas behind nozzles, $^{\circ}R$

$T_{g,3}$ static temperature of gas downstream of rotor, $^{\circ}R$

In some turbines it will probably be impractical, if not impossible, to place instruments at station 2. In this case, the temperature of the combustion gas must be calculated from conditions upstream of the nozzles or the condition at station 2 for specified static pressures will have to be calibrated with the rotor removed with reference to a measuring station upstream of the nozzles.

In calculating the static conditions and velocity of the gas at station 2 from the measured values at station 1 for the case of uncooled nozzles, it is assumed that the total temperature $T'_{g,2}$ and the total pressure $p'_{g,2}$ downstream of the nozzles are equal to the total temperature $T'_{g,1}$ and the total pressure $p'_{g,1}$ upstream of the nozzles. The assumptions are thought to be valid inasmuch as only a small heat loss to the casing is neglected. If the nozzles are cooled, a correction of the total temperature at station 2 must be made to allow for the heat added to the coolant. The heat added to the cooling air flowing through the nozzles is determined from the formula

$$Q_S = w_{a,S}\, c_{p,a,av} \left(T'_{a,T,S} - T'_{a,h,S}\right) \tag{15}$$

The total temperatures of the air in the nozzle cooling passage and the cooling-air flow passing through the nozzles are determined from measurements. The total temperature of the gas at station 2 is then obtained from

$$T'_{g,2} = T'_{g,1} - \frac{Q_S}{w_g c_{p,g}} \tag{16}$$

The specific heat of the gas at this point is determined from $T_{g,1}$ because $T_{g,2}$ is being calculated. If after $T_{g,2}$ is calculated $c_{p,g}$ based on the average of $T_{g,1}$ and $T_{g,2}$ is

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much different from the $c_{p,g}$ used at first, a recalculation of $T_{g,2}$ should be made using the $c_{p,g}$ based on the average temperature. With cooled nozzles, the assumption of $p'_{g,2}$ equal to $p'_{g,1}$ is thought to be more valid than before inasmuch as the cooled nozzles tend to stabilize the boundary layer and reduce the blading losses.

The method of calculating static conditions behind the nozzles [knowing $T'_{g,2}$, $p'_{g,2}$, $w_g$, and $A_2$ (the nozzle-outlet area)] is primarily a trial-and-error solution. With two additional assumptions, however, the nozzle-outlet conditions can be quite closely approximated. The following equation is developed in reference 6 (equation (9)):

$$\left[\left(\frac{p_{g,2}}{p'_{g,2}}\right)^{\frac{2}{\gamma_g}} - \left(\frac{p_{g,2}}{p'_{g,2}}\right)^{\frac{\gamma_g+1}{\gamma_g}}\right]^{\frac{1}{2}} = \frac{w_g}{CA_2\sqrt{\frac{2\gamma_g g}{(\gamma_g-1)R_g}}\;\frac{p'_{g,2}}{\sqrt{T'_{g,2}}}} \qquad (17)$$

where $C$ is the nozzle-discharge coefficient. With the nozzle-discharge coefficient assumed equal to 0.98, all the terms on the right-hand side of equation (17) are known from turbine measurements and the assumptions made, with the exception of the ratio of specific heats $\gamma_g$. For the first trial, the ratio is based on $T'_{g,2}$. After equation (17) has been solved for the pressure-ratio term (the left-hand side of equation (17)), the pressure ratio is obtained by making use of one set of curves in figure 3. When the pressure ratio is known, the temperature ratio $T_{g,2}/T'_{g,2}$ is obtained from the other set of curves in figure 3, which are based on the simple formula

$$\frac{T_{g,2}}{T'_{g,2}} = \left(\frac{p_{g,2}}{p'_{g,2}}\right)^{\frac{\gamma_g-1}{\gamma_g}} \qquad (18)$$

If the value of $T_{g,2}$ is appreciably smaller than $T'_{g,2}$, a new $\gamma_g$ based on the calculated $T_{g,2}$ is used and the procedure repeated. With the calculated value of $T_{g,2}$ and the value of $T_{g,3}$ obtained by normal means from measurements at station 3, the average static temperature can be found. (See equation (14).)

The foregoing method applies to flows up to a Mach number of 1.0 at the nozzle outlet and for no further expansion past this point. The case of supersonic flow caused by an expansion downstream of the nozzles is not considered herein.

Average total gas temperature $T''_{g,av}$. - The relative total gas temperature $T''_{g,av}$ can be evaluated from the average static gas temperature $T_{g,av}$ and the relative average rotor-inlet and -outlet gas velocities, $W_{2,av}$ and $W_{3,av}$, respectively. The expression used is

$$T''_{g,av} = T_{g,av} + \frac{\frac{W_{2,av} + W_{3,av}}{2}}{2Jgc_{p,g}} \tag{19}$$

Velocity diagrams, such as shown in figure 4, are made for four or five sections along the rotor-blade span and values of the inlet and outlet velocities are integrated to obtain the averages required in equation (19).

The velocity diagrams at any section are constructed as follows: When the turbine-rotor speed and the radius to the section being considered are known, the tangential velocity $u$ of the blade can be evaluated. (See fig. 4.) The angle $\alpha_2$ is known by assuming that the gas leaves the nozzles in the direction of the nozzle-outlet angle or by making surveys downstream of the nozzles with the rotor removed. The velocity $V_2$ can be calculated using measured values of $T_{g,2}$ and $T'_{g,2}$ or by using those calculated by methods previously given and the formula

$$V_2 = \sqrt{2Jgc_{p,g}\, T_{g,2} \left(\frac{T'_{g,2}}{T_{g,2}} - 1\right)} \tag{20}$$

The nozzle velocity coefficient was considered in the evaluation of $T_{g,2}$. The velocity $W_2$ can then be determined. (See fig. 4.) The velocity $V_3$ and direction angle $\alpha_3$ are determined by survey measurements just downstream of the rotor.

Prandtl and Mach numbers of gas. - The Prandtl number of the gas to use in equation (12) when determining the formula for variation of the recovery factor $\Lambda_g$ from experiments is based on the average static temperature $T_{g,av}$. The Mach number $M_{g,3}$, which has in the past generally led to good correlation of $\Lambda_g$ for static cascades, is calculated from the values of average relative velocity at the turbine outlet $W_{3,av}$, determined as described in the previous section, and the static gas temperature $T_{g,3}$. The formula is

$$M_{g,3} = \frac{W_{3,av}}{\sqrt{\gamma_g g R_g T_{g,3}}} \tag{21}$$

If surveys at station 3 are not feasible, the assumptions of $\alpha_3$ equal to rotor-blade-outlet angle and total conditions at station 3 equal to total conditions at station 4 provide means for calculating $V_3$ in order to get $W_3$ and $T_{g,3}$, which are needed to get $T''_{g,av}$ and $M_{g,3}$. If the mixing losses are large, however, this procedure might lead to possible errors.

Method of presenting $H_o$ data. - For the case of experiments with cold air flowing around a static cascade of heated impulse blades, it is determined in reference 4 that the outside heat-transfer coefficients could be correlated for design angle of attack in the form of the following equation:

$$\frac{\left(\dfrac{H_o \dfrac{l_o}{\pi}}{k_g}\right)}{\left(\dfrac{c_{p,g}\,\mu_g\,g}{k_g}\right)^{\frac{1}{3}}} = f^{II}\left(\frac{\rho_g V_g \dfrac{l_o}{\pi}}{\mu_g}\right) \tag{22}$$

where $l_o/\pi$ was the outside perimeter of the blade divided by $\pi$. The properties of the fluid, $c_{p,g}$, $\mu_g$, and $k_g$ were determined at a film temperature that is equal to one-half the sum of the average blade-wall temperature and the static temperature of the fluid immediately upstream of the blade leading edges. The density and the velocity of the fluid, $\rho_g$ and $V_g$, respectively, were determined from temperature and pressure measurements immediately upstream of the blade leading edge. The differences between the wall and stream temperatures were not very large.

Further unpublished investigations have been conducted on this cascade at design angle of attack using hot air (up to 300° F) and blade cooling. The difference between the wall and stream temperatures again was small. The data were worked up in the form given by equation (22) using film temperature, and so forth. The results when plotted gave a different line than the one obtained with cold air. Theory and experiments have indicated that if film temperature is used for correlating heat-transfer results, another factor in addition to Reynolds number and Prandtl number must be included in the equation. This factor is the ratio of the wall temperature to the static temperature of the fluid outside the fluid boundary layer on the surface (references 7 to 9).

It is evident from experiments (references 8 and 9) and theory (reference 7) that if the properties of the fluid are based on the wall temperature rather than the film temperature and, in addition, according to reference 9, if the density $\rho$ is also based on the wall temperature, correlation of data can be obtained over wide ranges of the ratio of wall to stream temperatures with the type of formula expressed by equation (22). This method was used for the case of hot- and cold-air experiments on the cascade of blades previously mentioned and a much better correlation of data was obtained than for the case where fluid properties were based on film temperature and the density was that determined from fluid measurements immediately upstream of the leading edges of the blades.

A correlation of six sets of data obtained on cascades of turbine blades whose profiles were vastly different and with the flow at the design angle of attack has been attempted at the Lewis laboratory. The data of reference 10 indicated that using the wall temperature for properties including the density and a formula of the form of equation (22) was insufficient to obtain satisfactory correlation. It was found that a better correlation could be obtained if a formula of the type

$$\frac{Nu}{(Pr)^{1/3}} = f^{III}\left(Re, \frac{T_g}{T_B}\right) \tag{23}$$

were used. A temperature-ratio effect is indicated by theory (reference 11). It is therefore suggested that a type of formula like equation (23) be used in attempting to correlate the outside heat-transfer-coefficient data obtained in turbine experiments, or,

$$\frac{\left(\frac{H_o \frac{l_o}{\pi}}{k_g}\right)}{\left(\frac{c_{p,g}\, \mu_g\, g}{k_g}\right)^{\frac{1}{3}}} = f^{IV}\left(\frac{\rho_g V_g \frac{l_o}{\pi}}{\mu_g}, \frac{T_{g,av}}{T_{B,av}}\right) \tag{24}$$

where the properties of the fluid and the density are based on wall temperature.

The foregoing attempts have been for design angle of attack of the gas flow. Changes in angle of attack of the fluid changes $H_o$ appreciably (reference 12). Thus the function $f^{IV}$ in equation (24) changes with each angle of attack and formulas like equation (24) must be established for each angle.

The use of blade-wall temperatures to determine factors in the heat-transfer-coefficient equation necessitates locating thermocouples in the walls of the blades of the turbine. Usually such instrumentation is kept to a minimum but enough thermocouples should be used to give a fair average for the wall temperature.

It is unfortunate that when analyzing the performance of the turbine for conditions other than those specifically tested, the blade-wall temperature must be assumed in order to calculate the heat-transfer coefficients from experimentally determined formulas. (The foregoing statement, of course, assumes that only by using blade temperatures can the data be correlated, which may not prove to be true in all future cases.) When the heat-transfer coefficient is calculated, the blade temperature distribution is calculated according to equation (3). If the calculated average blade temperature does not agree with the assumed value, then the procedure must be repeated using the new value. The results do not have to check exactly inasmuch as $T_{B,av}$ can vary somewhat without appreciably changing $H_o$. The heat-transfer coefficient also can vary somewhat without greatly affecting the results of equation (4).

In summary, for a cooled turbine it is suggested that experiments be conducted to establish a formula for each angle of attack, the form of which should be similar to equation (24), which it is expected will correlate the values of blade outside convection heat-transfer coefficients calculated according to equation (4). The properties of the fluid, $c_{p,g}$, $\mu_g$, $\rho_g$, and $k_g$ should be based on the average blade temperature. The velocity $V_g$ in the case of the rotor heat-transfer coefficients is replaced with the average relative velocity. This average relative velocity is equal to one-half the sum of $W_{2,av}$ and $W_{3,av}$, the methods for determining both of which have been given. The density should be based also on the average static pressure of the gas, which is one-half the sum of $p_{g,2}$ and $p_{g,3}$.

The static pressures of the combustion gas at the various stations is determined by instruments located radially and circumferentially so that a mass average can be found.

The use of $l_o/\pi$ for the characteristic dimension in equation (24) is suggested for the present. For turbines with low solidities, the blade may act like an isolated airfoil; whereas with high solidities, a dimension such as hydraulic diameter of the passage between the blades will result in correlation of data from one turbine with those from another. In any one turbine, however, it is immaterial which dimension is used inasmuch as it is a constant and can be included in the function $f$ of equation (24) rather than in the Nu and Re terms.

## Inside Convection Heat-Transfer Coefficient

Method of calculation. - The convection coefficient for the transfer of heat from the blades to the cooling air is calculated from data obtained in experiments on the heat gained by the cooling air, the average blade temperature, and the average effective air temperature in addition to the blade dimensions.

For hollow blades such as those shown in figures 5(a) and 5(b), the coefficient is calculated using the formula

$$H_i = \frac{Q_a}{l_i b\left(T_{B,av} - T_{a,e,av}\right)} \qquad (25)$$

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If an insert is used, such as in figure 5(c), it is assumed that radiation is negligible and that the insert adds little heat to the cooling air. Consequently the temperature of the insert is neglected and equation (25) is applicable to such a blade configuration.

When fins are used in a hollow blade, as shown in figure 5(d), the heat-transfer coefficient $H_i$ in equation (25) should be based on the average of the blade-wall and fin temperatures and the total surface in contact with the air rather than on the average wall temperature and wall surface area, as in equation (25). Inasmuch as it is difficult to instrument turbine blades with thermocouples, it would be advantageous if thermocouples could be used only in the blade walls and that $H_i$, based on average wall and fin temperature and the total surface area, could be obtained from a coefficient based on wall conditions only. For a finned blade such a relation exists. From reference 13, then, it can be shown that for the finned blade the following relation is true:

$$H_f = \frac{H_i}{m + \tau}\left[\frac{2 \tanh (\phi L_f)}{\phi} + m\right] \tag{26}$$

where

$$\phi = \sqrt{\frac{2H_i}{k_B \tau}}$$

In equation (26), the coefficient $H_f$ is based on wall conditions and is determined in a manner similar to equation (25).

It is necessary for correlation purposes to determine the coefficient $H_i$ based on average wall and fin temperatures. Because it is difficult to solve for $H_i$ in equation (26), it is advantageous to plot $H_i$ against $H_f$ and then for any value of $H_f$, determined from an equation similar to equation (25), $H_i$ can be found directly.

Average effective cooling-air temperature $T_{a,e,av}$. - The method of determining the average effective cooling-air temperature is very similar to that previously described for the effective gas temperature. The effective cooling-air temperature is defined as the adiabatic wall temperature of the blade and is related to the average total and static air temperatures by the expression

$$1 - \Lambda_a = \frac{T''_{a,av} - T_{a,e,av}}{T''_{a,av} - T_{a,av}} \qquad (28)$$

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where $\Lambda_a$ is the recovery coefficient of the blade on the cooling-air side.

As in the case of the recovery coefficient on the outside of the blade, a relation between cooling-air Mach and Prandtl numbers and $\Lambda_a$ must be established from experiments conducted with conditions as nearly adiabatic as possible; that is, no gas is passed around the outside of the blades. The average blade temperature is inserted in equation (28) for $T_{a,e,av}$ to determine $\Lambda_a$ for these experiments. A relation of the form

$$\frac{\Lambda_a}{\sqrt{Pr_a}} = f^V(M_a) \qquad (29)$$

will probably correlate the data. The effective cooling-air temperature $T_{a,e,av}$, used to determine $H_i$ from the heat-transfer experiments, is then calculated knowing the cooling-air conditions; namely, $Pr_a$, $M_a$, $T''_{a,av}$, and $T_{a,av}$ for these heat-transfer experiments and the relations given by equations (28) and (29). Consequently, in order to solve for $T_{a,e,av}$, methods must be known for evaluating these four cooling-air conditions.

*Average static cooling-air temperature* $T_{a,av}$. - It is desirable to have temperature and pressure measurements at a number of spanwise stations along the blade in the cooling-air passages in order to have an accurate picture of the conditions of the cooling air. It is assumed herein, however, that measurements are made only at the blade root and tip, which is usually more practical. Instruments should be so located in the blade passages at these two stations that accurate average measurements over the cross section can be obtained. Because the instruments are rotating with the blades, all total values calculated from the measurements are relative to the blades as previously mentioned.

The effect of heat transfer, friction, and rotation on the static-temperature distribution of the cooling air in the blade passage is such that the variation of this static temperature can be assumed linear if the Mach number of the cooling air is low, say below 0.5. Thus,

$$T_{a,av} = \frac{T_{a,h} + T_{a,T}}{2} \qquad (30)$$

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The static cooling-air temperatures at the root and the tip, $T_{a,h}$ and $T_{a,T}$, are determined by normal methods from the thermocouple measurements at these stations and the recovery factors of the particular instruments that are used.

If the Mach number reaches high values, the static-temperature variation with the blade-length parameter $y$ will probably be similar to that indicated in figure 6 and the assumption of linear variation is not quite valid. In order to accurately obtain the average static temperature of the cooling air under such conditions, a differential equation must be solved that involves the Mach numbers and the total temperatures at the root and the tip. The method of determining the values of total temperature mentioned have been described in connection with equation (10). The differential equation and its solution to determine the variation of Mach number and total temperature through the cooling-air passages is the main subject of reference 3. With the solution of the differential equation, the static temperature at any point $x$ along the blade span is calculated from the expression

$$T_{a,x} = \frac{T''_{a,x}}{1 + \frac{\gamma_a - 1}{2} M_{a,x}^{2}} \tag{31}$$

The average temperature $T_{a,av}$ is determined by integrating the values of $T_{a,x}$ with respect to the cooling-passage length, or blade span.

Average total cooling-air temperature $T''_{a,av}$. - The total cooling-air temperature is practically linear with respect to blade length for all Mach numbers and the average can be determined as the arithmetic mean of $T''_{a,h}$ and $T''_{a,T}$, which are the same values used in equation (10).

Cooling-air Mach numbers. - The cooling-air Mach numbers at the blade root and tip can be calculated from the total and static pressures at these stations obtained from measurements by methods previously given. The correct Mach number to use in equation (29) to correlate the recovery-factor data is unknown at present but it is suggested that an average value for the cooling-air passages be used. For low Mach numbers, an arithmetic mean of $M_{a,h}$ and $M_{a,T}$ is thought to be sufficient. For high Mach numbers, an integrated average of the values determined by means of equation (1) of reference 3 is suggested.

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Cooling-air Prandtl number. - The cooling-air Prandtl number to use in equation (29) should be based on the average static cooling-air temperature $T_{a,av}$.

Method of presenting $H_i$ data. - No data on the convection heat-transfer coefficient from blades to cooling air in turbine-blade cooling passages using either cascades of blades or turbines are available. Consequently, no generalized form of a formula similar to equation (23) has been experimentally verified for these coefficients. The flow of cooling air through the turbine passages, however, can be thought of as flow of a fluid through a tube where the tube may have shapes such as those shown in figure 5. As a consequence, it would be expected that the inside coefficient data could be correlated by means of a formula similar in form to that which is known to correlate the experimental heat-transfer data for fluids flowing through tubes or pipes. The form of this formula for pipes is (reference 14, p. 164)

$$Nu = f^{VI}(Re,Pr) \tag{32}$$

A discussion is given in reference 1 on factors that may influence the heat-transfer coefficient of turbine-blade cooling passages. Effects of fluid flow at the passage inlet, which may cause $H_i$ to vary along the blade span, effect of an annulus shape, such as in figure 5(c), on the function $f^{VI}$ in equation (32), and other details are included. On the basis of this discussion, it is suggested that for the present, in accordance with equation (32), experiments should be conducted to determine if an equation similar to the following can be used to correlate the inside heat-transfer coefficients:

$$\frac{\left(\dfrac{H_i D_{h,a}}{k_a}\right)}{\left(\dfrac{c_{p,a}\mu_a g}{k_a}\right)^n} = f^{VII}\left(\frac{\rho_a W_a D_{h,a}}{\mu_a}\right) \tag{33}$$

The function $f^{VII}$ is determined from the investigations.

It is suggested on the basis of pipe experiments that an exponent $n$ of 0.4 be used in equation (33). The hydraulic diameter $D_{h,a}$ of the cooling passage to be used in equation (33) is defined by the table in figure 5 for the various blade-passage configurations.

It is suggested, on the basis of results in reference 9, that the fluid properties, including density, in equation (33) be based on average wall temperatures $T_{B,av}$. Because, as discussed in reference 1, analysis of data at inlet sections to pipes shows marked variation of heat-transfer coefficient with distance from the inlet, use of surface temperature for determining properties may not correlate the data for the coefficients in the blade passages where inlet effects may predominate. The use of surface temperatures was satisfactory for the long tubes used in reference 9 and is suggested as a first trial in attempts to correlate blade inside coefficients.

The density should also be based on the average static pressure in the coolant passage. For low Mach numbers, this average can be the arithmetic mean of the pressures at the blade root and tip. The methods for determining the blade-tip pressures have been given. For high Mach numbers, the average static pressure must be obtained by integrating calculated values determined as follows: Values of Mach number and static temperature are known through the passage by methods previously described. The relative velocity at each radial station is determined from

$$W_{a,x} = M_{a,x}\sqrt{\gamma_a g R_a T_{a,x}} \tag{34}$$

The static pressure at each radial station is then calculated from the expression

$$p_{a,x} = \frac{w_a R_a T_{a,x}}{W_{a,x} A_{a,x}} \tag{35}$$

For low Mach numbers, the velocity $W_a$ to be used in equation (33) is an arithmetic mean of velocities at the blade root and tip determined from measurements; for higher Mach numbers, $W_a$ is an integrated average of the velocities determined along the passage by means of equation (34). All terms are thus determined so that the three parameters, $Nu_a$, $Pr_a$, and $Re_a$ of equation (33) can be calculated from the experimental data.

The suggested form of formula as given by equation (33) is based on the premise that the heat transfer from the blade to the coolant is due to forced convection. For heat transfer by forced convection alone, Reynolds and Prandtl numbers are the only significant parameters affecting heat transfer. Forced convection is

expected to be the predominant type of heat transfer for cooling air flowing through blade passages that are open at both ends and are of the type used in air-cooled turbines. As a result, the use of equation (33) is suggested as a first trial in attempts to correlate blade inside heat-transfer coefficients.

Natural convection heat transfer has been studied through the use of water-cooled turbines. The coolant passages in the blades are holes that are plugged at the tip end of the blade. A circulatory action of the coolant takes place, the coolant flowing from root to tip through the center of a hole and from tip to root along the outside of the same hole. A complete discussion of the governing parameters for heat transfer for such a flow is given in reference 1. With this type of circulation, the heat transfer is a natural-convection phenomena and in any such phenomena the Nusselt number is a function of Grashof and Prandtl numbers (references 14 and 15).

The possibility exists that, even in through coolant passages as in air-cooled blades, a small circulatory motion of the air in the passage may be superimposed on the main fluid motion, which is radial. This possibility may be more true for the case of hollow blades with large passages than for finned blades in which the passages are small tubes. Consequently, even with air-cooled blades of the form discussed (that is, with through passages), a small transfer of heat by natural convection in addition to a large transfer of heat by forced convection may occur. If the data do not correlate by a formula of the form of equation (33), then a form as follows may be applicable:

$$Nu = f^{VIII}(Gr,Re,Pr) \tag{36}$$

Experiments must then be conducted over a range of both Grashof and Reynolds numbers to determine the function $f^{VIII}$.

The Grashof number for the turbine blade is calculated using the expression

$$Gr_a = \frac{D_{h,a}{}^3 \rho_a{}^2 g' \beta_a \left(T_{B,av} - T_{a,av}\right)}{\mu_a{}^2} \tag{37}$$

The gravity term $g'$ is many times greater than the ordinary gravity term $g$ (reference 1) and varies with speed. This gravity term equals $\omega^2 r$, where $\omega$ is the angular velocity of the

turbine wheel and $r$ is the radius from the center of rotation to the point considered. As a consequence $g'$ varies from root to tip. It is suggested that $g'$ be determined at a mean radius that is a geometric mean of the root and tip radii, $\sqrt{r_h r_T}$.

The coefficient of thermal expansion $\beta_a$ generally is expressed by the formula

$$\beta = \frac{dv/dT}{v_{initial}} \tag{38}$$

where $dv/dT$ is the change in specific volume with temperature and $v_{initial}$ is the initial specific volume. Then equation (38) becomes for the cooling air in the blade passage,

$$\beta_a = \rho_{a,h}\left(\frac{\dfrac{1}{\rho_{a,h}} - \dfrac{1}{\rho_{a,T}}}{T_{a,h} - T_{a,T}}\right) \tag{39}$$

This equation reduces to

$$\beta_a = \frac{1}{T_{a,h}}\left(\frac{T_{a,h} - \dfrac{p_{a,h}}{p_{a,T}} T_{a,T}}{T_{a,h} - T_{a,T}}\right) \tag{40}$$

because, for small pressure changes, $\beta$ is approximately equal to $1/T$. For small pressure changes in the turbine cooling passage, that is, $p_{a,h}/p_{a,T} \approx 1$, equation (40) reduces to $\beta_a \approx 1/T_{a,h}$. The methods of determining the air pressures and temperatures in equation (40) have been given.

The density $\rho_a$, the viscosity $\mu_a$, and the hydraulic diameter $D_{h,a}$ in equation (37) can be treated in the same manner as suggested for the Reynolds number determination (equation (33)).

## Miscellaneous Factors

In giving the suggested methods for determining and correlating the outside and inside convection heat-transfer coefficients, $H_o$ and $H_i$ from turbine investigations, methods for determining various

factors required to establish the validity of equation (3), which is part of any turbine-investigation program, have also been given. Methods have not been given, however, for obtaining $T_{B,x}$ and $T_{a,e,h}$ in equation (3) or various specific-heat and ratio-of-specific-heats terms in several equations. These terms are now given.

Effective cooling-air temperature at blade root $T_{a,e,h}$. - The method of determining the effective cooling-air temperature at the blade root is the same as that for determining the average effective cooling-air temperature $T_{a,e,av}$, except all values of Mach number, Prandtl number (used to get recovery factor at blade root), blade temperature, and so forth are the values obtained from the data at the blade root. The investigations for $\Lambda_{a,h}$ can be simultaneously conducted with those for the average recovery factor inside the blade. The effective air temperature used in the equation to determine the variation of $\Lambda_{a,h}$ with Prandtl and Mach numbers is the integrated average of the peripheral blade temperatures at the root.

Average blade temperature at any spanwise station x, $T_{B,x}$. - The average blade temperature at any position $x$ $T_{B,x}$ is the integrated average of the blade-temperature readings at this position as obtained from the data. These values are compared with $T_{B,x}$ calculated using equation (3) to verify the use of this equation. Such averages have been mentioned as the first step required in the determining of $T_{B,av}$.

Average specific heat of cooling air in rotor blades. - An average specific heat of the cooling air is required in equation (10) to determine the heat gained by the air in the rotor-blade cooling passages. It is suggested that this specific heat be determined at an arithmetic average of the root and tip cooling-air static temperatures. This average should be accurate enough for all practical purposes.

Average specific heat of cooling air in nozzle blades. - If the nozzle blades are cooled, the heat gained by the air passing through them is required. (See equation (15).) The specific heat in this equation can be determined in the same manner as suggested for $c_{p,a,av}$ for the rotor blades, except that temperature measurements made in the nozzle-blade cooling-air passages are used.

<u>Over-all convection heat-transfer coefficient U.</u> - A useful factor heretofore unmentioned is the over-all convection heat-transfer coefficient $U$. The heat transfer from the gas to the cooling air can be expressed in terms of the over-all coefficient as follows:

$$Q_a = U l_o b \Delta T_e \tag{41}$$

where $\Delta T_e$ is the mean effective temperature difference between the gas and the cooling air. The over-all coefficient is useful in that if its variation with gas and coolant conditions is established, calculations of turbine performance at conditions other than those of the experiments are simplified. Also a check of the individual coefficients, $H_o$ and $H_i$, is obtained. Both of these objectives are obtained if the over-all coefficients calculated by means of equation (41) agree with coefficients calculated from the formula

$$U = \frac{\frac{1}{l_o}}{\frac{1}{H_o l_o} + \frac{1}{H_i l_i}} \tag{42}$$

where $H_o$ and $H_i$ are calculated from formulas established as previously described from experiments on the turbine (equations (4) and (25)). Equation (42) is valid with the assumption that radiation and temperature drop through the blade wall are negligible. It is recommended that values of $U$ for all experiments made on a turbine be calculated by the two procedures and compared. Agreement will indicate that the assumptions of equation (42) are valid. If such agreement exists, it is also recommended that comparison of values of $U$ using equation (42) with $H_o$ and $H_i$ values calculated from the formulas that correlate the data, for example, equation (24), be made with values calculated using the experimental data in equation (41). Such a comparison will show the error involved in using $H_o$ and $H_i$ correlation formulas to calculate $U$ for conditions other than those tested. If the error is small, $U$ calculated in this manner can be used to calculate $Q_a$ for such conditions using equation (41). The heat gained by the cooling air $Q_a$ is required in performance evaluations.

It may be possible to obtain a formula that correlates the over-all coefficients determined from the experimental data.

Because of the form of equation (42) and because $H_o$ and $H_i$ in turn are functions of several air and gas parameters, this formula may be complicated. If the values of $U$ can be correlated, the steps required to determine $Q_a$ for performance evaluations for other than experimental conditions will be decreased.

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Mean temperature difference from gas to cooling-air $\Delta T_e$. - The over-all coefficient $U$ is based on a temperature difference $\Delta T_e$ (equation (41)), which is a mean of the temperature differences between the gas and the cooling air. This temperature difference is called the logarithmic mean temperature difference. The general significance of this factor is discussed in reference 14.

In the case of the turbine rotor on the basis of equations (4) and (25), $\Delta T_e$ is

$$\Delta T_e = T_{g,e,av} - T_{a,e,av} = \frac{Q_a}{l_i b H_i} + \frac{Q_a}{l_o b H_o} \tag{43}$$

But from equation (41),

$$\Delta T_e = \frac{Q_a}{U l_o b}$$

Equation (42) can be obtained by equating equations (43) and (41).

Charts have been made (for example, reference 16) by means of which $\Delta T_e$ can be obtained for given inlet and outlet hot and cold fluid temperatures for many types of heat exchanger. As the air-cooled turbine can be considered to be roughly a cross-flow heat exchanger, it might be thought that the charts would be applicable. In deriving the equations for the charts, however, no distinction is made between effective, total, and static temperatures. The differential equations involve all such temperatures but each is replaced by the fluid temperature. For the rotating turbine, such an assumption can cause large errors and it is recommended that $\Delta T_e$ be determined from the difference between average effective gas and cooling-air temperatures, as shown by equation (43). Methods for determining these averages have been given.

### Stator-Blade Characteristics

The foregoing discussion has dealt mostly with methods for determining the rotor-blade heat-transfer characteristics. If the nozzles or stator blades are to be cooled, experiments should be conducted to determine like heat-transfer characteristics for these parts. The methods are similar, the only difference being that velocities and so forth relative to the stator-blade surface are equal to the values relative to the turbine casing; consequently, in formulas suggested for the rotor blades, such symbols as $W$, $T''$, and $p''$ are replaced by $V$, $T'$, and $p'$ and the formulas can then be applied to the stator blades. The measurements outside and inside the stator blades are used to establish the functions of the formulas where required.

Natural convection heat transfer (a Grashof number effect) is not considered in stator-blade characteristics.

## SUMMARY OF METHODS

As a brief résumé of the foregoing methods, the suggested primary formulas for the determination of the rotor-blade heat-transfer characteristics of an air-cooled turbine are presented. The formulas with few changes, which are explained in the preceding section, apply equally well to the stator blades.

In order to determine the coolant-flow requirements for safe operation at a given gas temperature for other than experimental conditions, the allowable and actual blade-temperature distribution must be calculated. The allowable-temperature curve is determined from stress-rupture data of the blade material, where the stress on a radial increment of the blade is calculated using equation (2). The actual blade-metal temperature distribution is found from equation (3) if this equation is proved valid in the turbine experiments. In this equation, the convection heat-transfer coefficients on both the outside and inside surfaces of the blades must be known. For a given turbine, these coefficients are determined from the experimental data.

The outside convection heat-transfer coefficient is determined from the data using equation (4). After values of $H_o$ have been evaluated, an attempt should be made to correlate these values in a dimensionless form similar to equation (24). It is suggested that the fluid properties including density be based on average blade-wall temperature. In addition, the density is based on the average

static pressure of the gas. The velocity $W_g$ in the Reynolds number is the average stream velocity in the blade passages, relative to the blade.

The inside coefficient $H_i$ is treated in a similar manner and is determined from the experimental measurements using equation (25). Correlation of the values of $H_i$ so determined should be attempted by the use of dimensionless parameters in the form of equation (33) or equation (36), depending on which is the more applicable for the turbine investigated. As a first trial, the fluid properties, including density, in the parameters, are based on average blade temperatures. The density is also based on average static pressure in the coolant passages. The velocity $W_a$ is the average stream velocity relative to the blade and can be determined from an integrated average of values along the passage calculated by means of equation (34). The Grashof number in equation (36) is determined using equation (37). The coefficient-of-thermal-expansion term in equation (37) is found by using equation (40).

The purpose of evaluating and attempting to correlate the over-all heat-transfer coefficient U is to have a check on the individual coefficients $H_o$ and $H_i$, and also to simplify turbine- and engine-performance evaluations. The over-all coefficients are determined from equations (41) and (42) and then compared to determine the error between $H_o$ and $H_i$ determinations and U determinations.

As previously mentioned, it is necessary to know the effective fluid temperatures, both of the combustion gases and the cooling air. The recovery factor $\Lambda$, which is an indication of the effective temperature, is found from experiments by using the following formula:

$$1-\Lambda = \frac{T''-T_e}{T''-T}$$

and can probably be correlated in a manner represented by

$$\frac{\Lambda}{\sqrt{Pr}} = f(M)$$

The mean temperature difference from the gas to cooling air $\Delta T_e$ was used in the determination of the over-all heat-transfer coefficient in equation (41) and is determined from equation (43).

The heat loss to the cooling air is the term on which the convection heat-transfer coefficients are directly dependent and can be calculated by using equation (10). The factor $f_s$ is determined by motoring the turbine wheel at several speeds with no gas passing across the blades and with various quantities of cooling air passing through the blades. In an experiment of this type $q_a = 0$, and $f_s$ can be readily determined.

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# APPENDIX A

## SYMBOLS

The following symbols are used in this report:

| | |
|---|---|
| $A$ | cross-sectional area, sq ft |
| $B$ | force, lb |
| $b$ | height of blade (span), ft |
| $C$ | nozzle-discharge coefficient |
| $c_p$ | specific heat at constant pressure, Btu/(lb)(°F) |
| $D$ | characteristic dimension in Reynolds and Nusselt numbers, ft |
| $D_h$ | hydraulic diameter $4A/l$, ft |
| $f_s$ | slip factor |
| $f^{I}$, $f^{II}$, $f^{III}$, $f^{IV}$, $f^{V}$, $f^{VI}$, $f^{VII}$, $f^{VIII}$ | unknown functions |
| $Gr$ | Grashof number $\frac{D^3\rho^2 g'\beta(T_B-T_e)}{\mu^2}$ |
| $g$ | ratio of absolute to gravitational unit of mass, lb/slug, or acceleration due to gravity, ft/sec$^2$ |
| $g'$ | acceleration used in Grashof number $(\omega^2 r)$, ft/sec$^2$ |
| $H$ | convection heat-transfer coefficient, Btu/(sec)(sq ft)(°F) |
| $J$ | mechanical equivalent of heat, 778.3 ft-lb/Btu |
| $k$ | thermal conductivity, Btu/(°F)(ft)(sec) |

| | |
|---|---|
| $L_f$ | effective width of fins from blade wall to mean camber line, ft |
| $l$ | perimeter, ft |
| M | Mach number |
| m | average spacing between fins in finned blade, ft |
| Nu | Nusselt number, $\left(\frac{HD}{k}\right)$ |
| n | exponent |
| Pr | Prandtl number, $\left(\frac{c_p \mu g}{k}\right)$ |
| p | static pressure, lb/sq ft absolute |
| p' | total pressure relative to turbine casing, lb/sq ft absolute |
| p" | total pressure relative to moving blade, lb/sq ft absolute |
| Q | heat flow, Btu/sec |
| q | heat flow per unit weight, Btu/lb |
| R | gas constant, ft-lb/(lb)($^{\circ}$F) |
| Re | Reynolds number, $\left(\frac{DW\rho}{\mu}\right)$ |
| r | radius, ft |
| S | stress, lb/sq in. |
| T | static temperature, $^{\circ}$R |
| $\Delta T_e$ | mean temperature difference effecting heat transfer, $^{\circ}$F |
| T' | total temperature relative to turbine casing, $^{\circ}$R |

| | | |
|---|---|---|
| $T''$ | | total temperature relative to moving blade, °R |
| $U$ | | over-all heat-transfer coefficient, Btu/(sec)(sq ft)(°F) |
| $u$ | | tangential velocity, ft/sec |
| $V$ | | absolute velocity, ft/sec |
| $v$ | | specific volume, cu ft/lb |
| $W$ | | relative velocity, ft/sec |
| $w$ | | weight flow, lb/sec |
| $X$ | = | $\dfrac{\omega^2 r_h w_a}{JgH_o l_o\left(T_{g,e} - T_{a,e,h}\right)}$ |
| $x$ | | radial distance from blade root to point on blade span considered, ft |
| $Y$ | = | $\dfrac{\omega^2 w_a^2 (\lambda + 1) c_{p,a}}{JgH_o^2 l_o^2\left(T_{g,e} - T_{a,e,h}\right)}$ |
| $y$ | | $\dfrac{r_T - r_x}{b}$ |
| $Z$ | = | $\dfrac{1}{\lambda + 1} \dfrac{H_o l_o b}{w_a c_{p,a}} \dfrac{x}{b}$ |
| $\alpha$ | | angle between absolute and tangential velocity vector, deg |
| $\beta$ | | coefficient of thermal expansion, 1/°F |
| $\gamma$ | | ratio of specific heats, $(c_p/c_v)$ |
| $\Lambda$ | | recovery factor |
| $\lambda$ | | $\dfrac{H_o l_o}{H_i l_i}$ |

| | |
|---|---|
| $\mu$ | absolute viscosity, slugs/(sec)(ft) |
| $\rho$ | density, slugs/cu ft |
| $\tau$ | thickness of fins in finned blades, ft |
| $\phi$ | $\sqrt{\frac{2H_i}{k_B \tau}}$ |
| $\Omega$ | work added to cooling air by blade rotation, Btu/(lb) |
| $\omega$ | angular velocity, radians/sec |

Subscripts:

| | |
|---|---|
| a | cooling air |
| at | atmospheric |
| av | average |
| B | blade |
| e | effective, used with symbol for temperature and denotes temperature effecting heat transfer |
| f | finned blades |
| g | combustion gas |
| h | blade root |
| i | inside surface of blade |
| o | outside surface of blade |
| S | stator blades |
| T | blade tip |
| t | pressure tube |
| x | point along blade span |

| | |
|---|---|
| 1 | station in combustion-gas stream at turbine inlet |
| 2 | station in combustion-gas stream between nozzles and rotor blades |
| 3 | station in combustion gas stream at rotor outlet |
| 4 | station in combustion gas stream downstream of rotor where swirl has largely disappeared |

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## APPENDIX B

## CORRECTION OF RECORDED MANOMETER PRESSURES OBTAINED WITH ROTATING TUBES

Consider the pressure equilibrium of a fluid element in a pressure line from the center line of the turbine to some radial point in the blade coolant passage (fig. 7). The pressure force directed outward on the fluid element is

$$B_1 = p_a A_t \tag{B1}$$

where

$p_a$ pressure of cooling air in tube, lb/sq ft absolute

$A_t$ cross-sectional area of tube, sq ft

The pressure force directed toward the center of rotation is

$$B_2 = \left(p_a + \frac{dp_a}{dr_t} \Delta r_t\right) A_t \tag{B2}$$

where

$\frac{dp_a}{dr_t}$ differential change of pressure in tube with respect to change of radial length, (lb/sq ft)/ft

$\Delta r_t$ radial increment of fluid considered, ft

The difference between these two forces gives a resultant inward force of $(dp_a/dr_t)\Delta r_t A_t$, which must be balanced by the centrifugal force on the element in order for equilibrium to exist. This centrifugal force $B_3$ can be given by the formula

$$B_3 = \left[\rho_{a,t} A_t \Delta r_t\right] \omega^2 r_t \tag{B3}$$

where

$\rho_{a,t}$ density of fluid in tube, slugs/cu ft

$\omega$ angular velocity of tube, radians/sec

$r_t$ radius to midpoint of element considered, ft

Because $B_2$ equals $B_1$ plus $B_3$, the following equation results from equations (B1) to (B3):

$$\frac{dp_a}{dr_t} = \rho_{a,t}\,\omega^2 r_t \tag{B4}$$

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When the fluid is compressible and the angular velocity is large, the density must be considered variable with pressure and temperature. Combining equation (B4) with the equation of state for a perfect gas gives

$$\frac{dp_a}{dr_t} = \frac{p_a}{gR_aT_{a,t}}\,\omega^2 r_t \tag{B5}$$

where

$R_a$ gas constant for air in tube, ft-lb/(lb)($^{\circ}$F)

$T_{a,t}$ temperature of air in tube, $^{\circ}$R

The temperature of the air $T_{a,t}$ will probably not be constant along the tube. The selection of the proper temperature is important inasmuch as a 100° F difference between the correct average temperature and the temperature used can in some cases cause an error in pressure of more than 1 pound per square inch. Thermocouples should be inserted in the pressure tubes over the length of the tube to find this average temperature. It is possible that the tubes if fastened to the rotor disk will assume the temperature of the disk and an average temperature of the disk will be adequate.

Equation (B5) can be put in the form

$$\frac{dp_a}{p_a} = \left[\frac{\omega^2}{gR_aT_{a,t}}\right] r_t\,dr_t$$

which after integration becomes

$$\log_e p_a = \frac{\omega^2 r_t^2}{2gR_aT_{a,t}} + K \tag{B6}$$

The integration constant K must be evaluated. When $r_t = 0$, K equals the logarithm of the pressure at the center line of the turbine, which is the value measured on the manometer board. This pressure is $p_{a,m}$, where m indicates measured. Then equation (B6) becomes

$$\log_e \frac{p_a}{p_{a,m}} = \frac{\omega^2 r_t^{\ 2}}{2gR_aT_{a,t}}$$

or

$$\frac{p_a}{p_{a,m}} = e^{\left(\frac{\omega^2 r_t^{\ 2}}{2gR_aT_{a,t}}\right)}$$

The manometer-board reading is the difference between the pressure $p_{a,m}$ and the pressure of the room in which the manometer board is placed or

$$\Delta p_{a,m} = p_{a,m} \pm p_{at}$$

The true pressure at any radius $r_t$ in the blade passage is then

$$p_a = \left(\Delta p_{a,m} \pm p_{at}\right) e^{\left(\frac{\omega^2 r_t^{\ 2}}{2gR_aT_{a,t}}\right)} \tag{B7}$$

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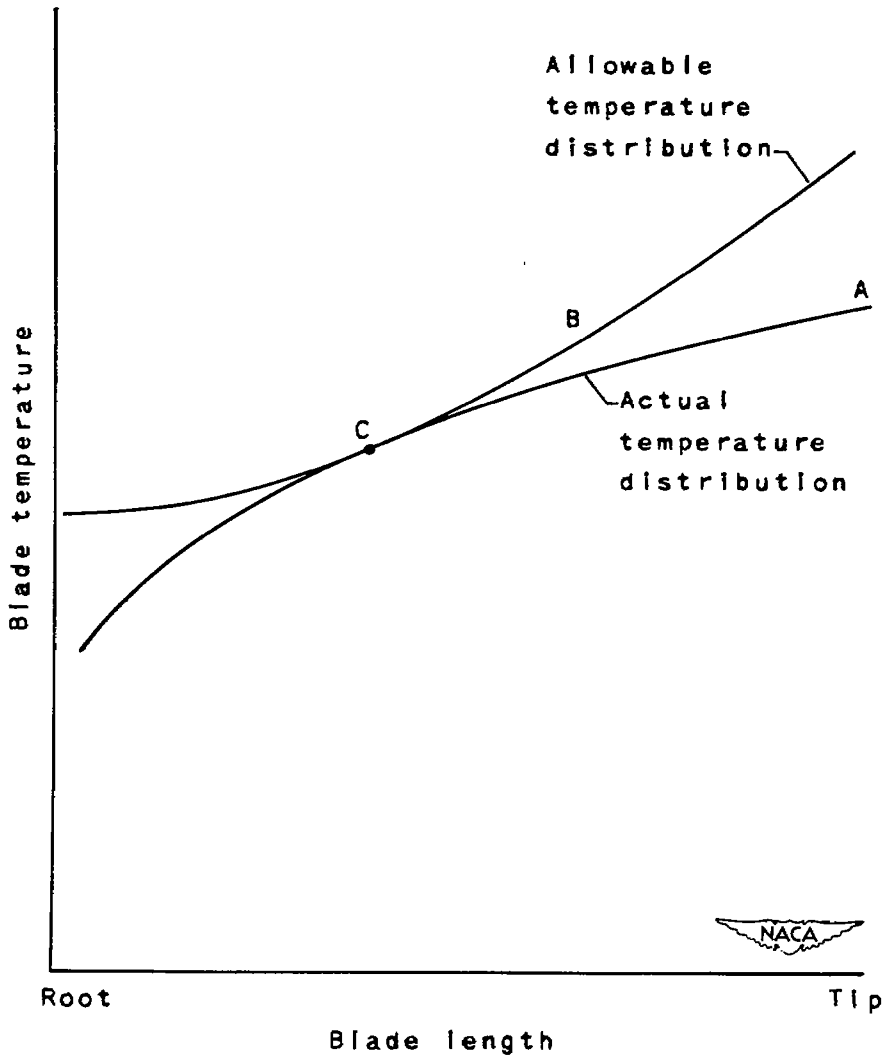
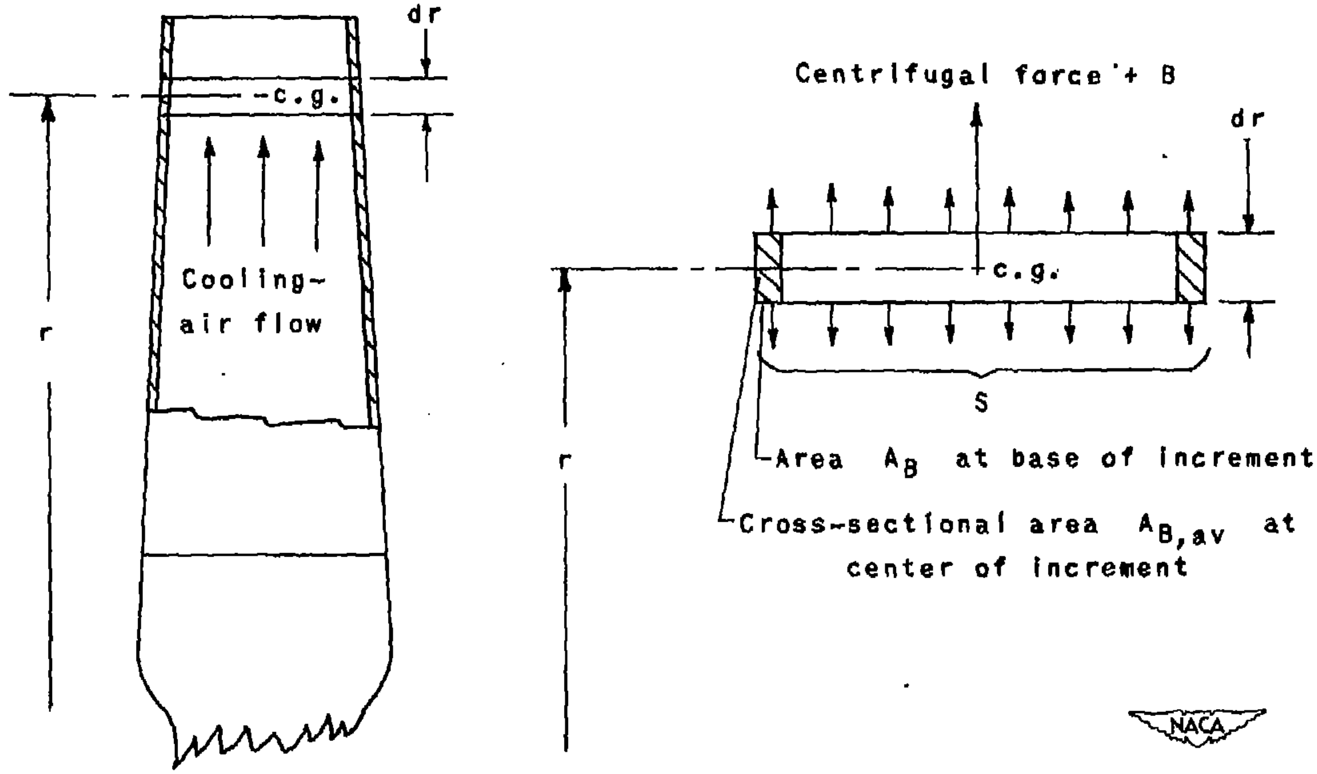



Figure 1. - Variation of allowable temperature distribution and actual temperature distribution with blade length for a minimum coolant flow.




(a) Cutaway view of typical turbine blade.

(b) Increment of turbine blade used in centrifugal-stress calculations.

Figure 2. - Typical air-cooled blade with increment to be used in centrifugal-stress calculations.

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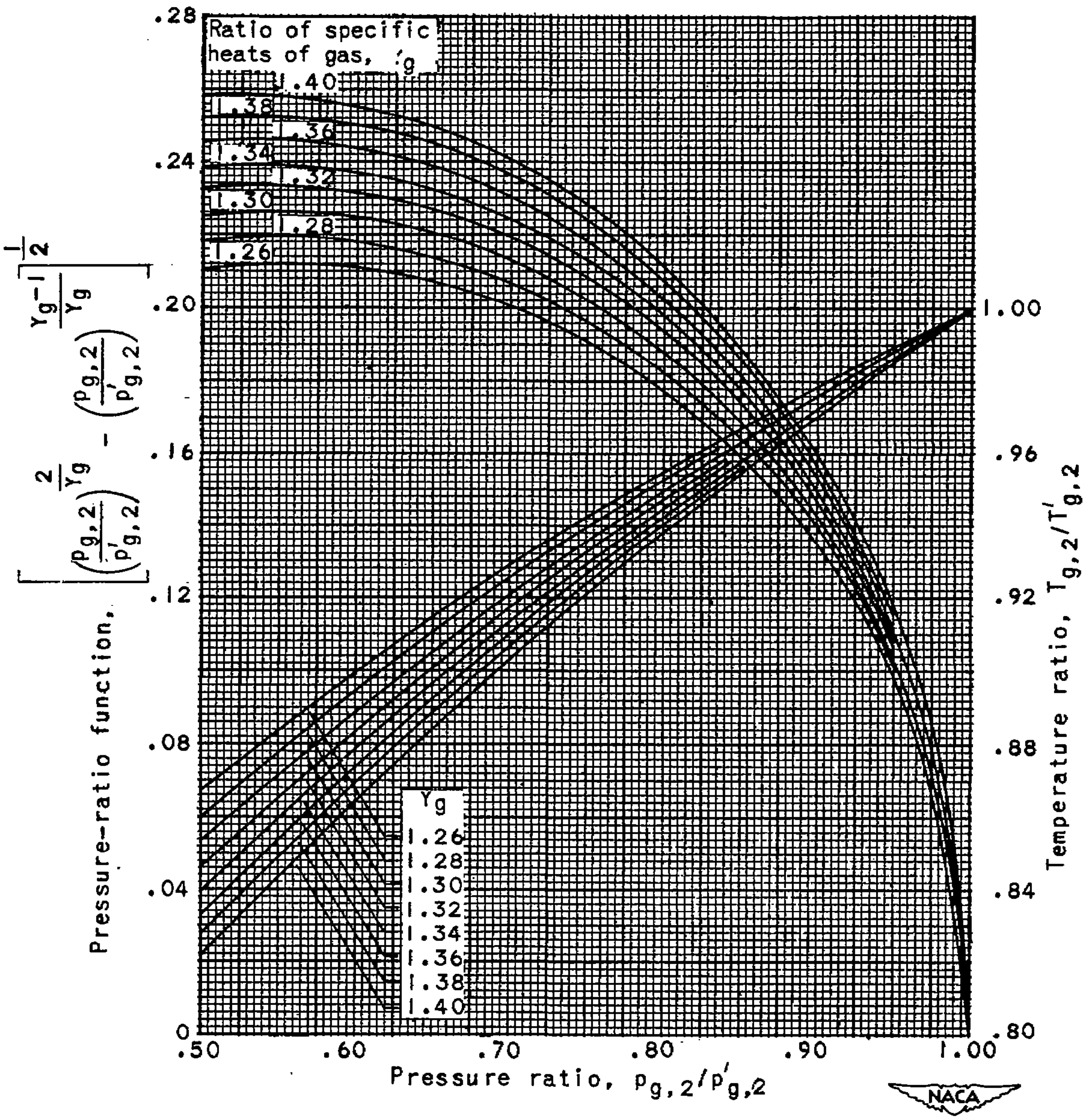


Figure 3. - Relation of pressure-ratio function, pressure ratio, and temperature ratio for several values of ratio of specific heats of gas.

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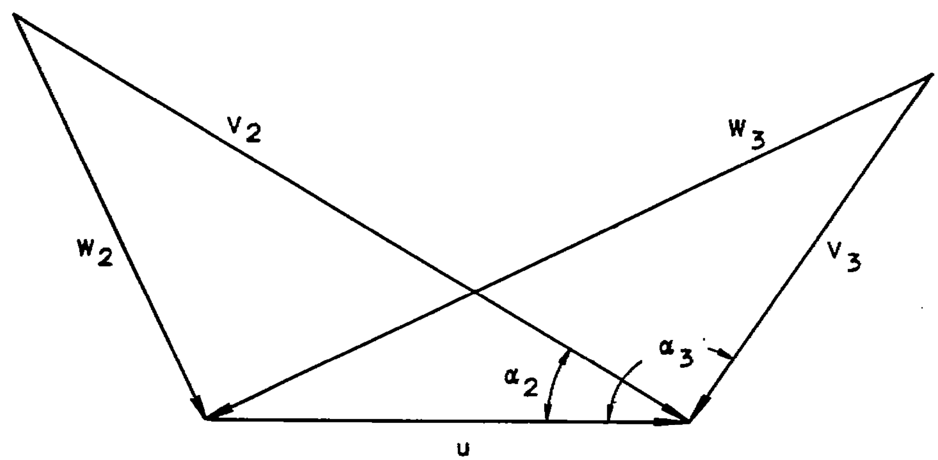


Figure 4. - Typical velocity diagram for a turbine blade.

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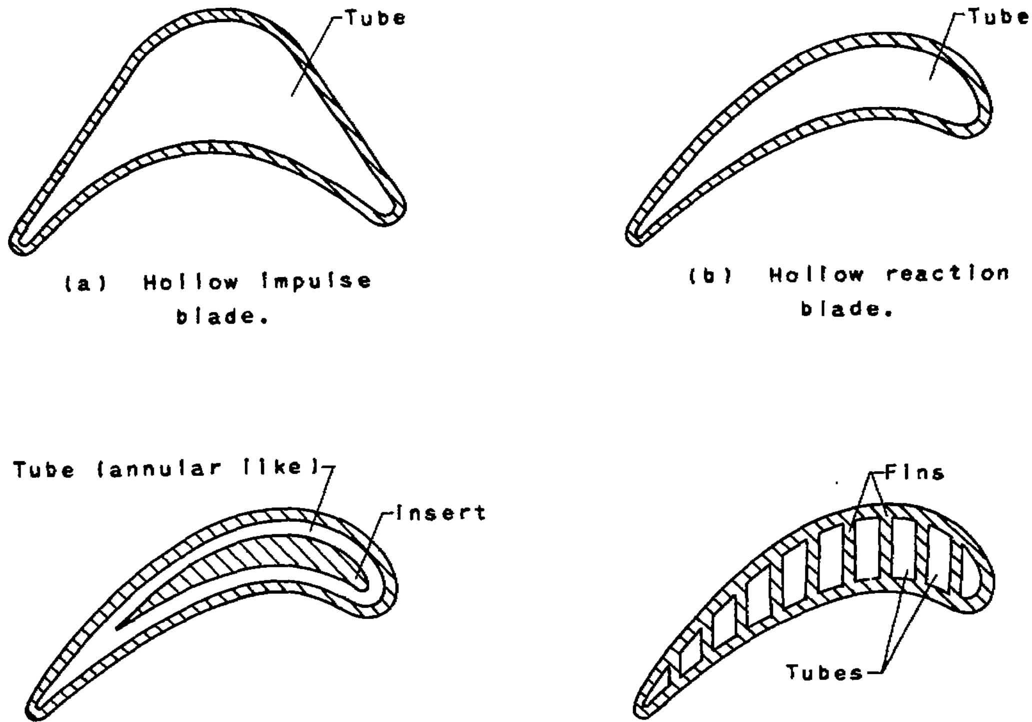



(a) Hollow impulse blade.

(b) Hollow reaction blade.

(c) Hollow blade with insert.

(d) Hollow blade with fins.

Determination of $D_{h,a}$

| | | |
|---|---|---|
| Hollow blade | $\dfrac{\text{4 × passage area}}{\text{wetted perimeter}}$ | $\dfrac{4A_{a,av}}{l_i}$ |
| Blade with insert | $\dfrac{\text{4 × passage area}}{\text{sum of inside blade perimeter and insert perimeter}}$ | $\dfrac{4A_{a,av}}{l_i + l_{insert}}$ |
| Finned blade | $\dfrac{\text{4 × summation of cross-sectional area of each passage}}{\text{summation of individual passage perimeters}}$ | $\dfrac{4\Sigma A_{a,av}}{\Sigma l_i}$ |

Figure 5. - Four typical air-cooled turbine-blade configurations and hydraulic diameter of cooling-air passage to be used in each case.

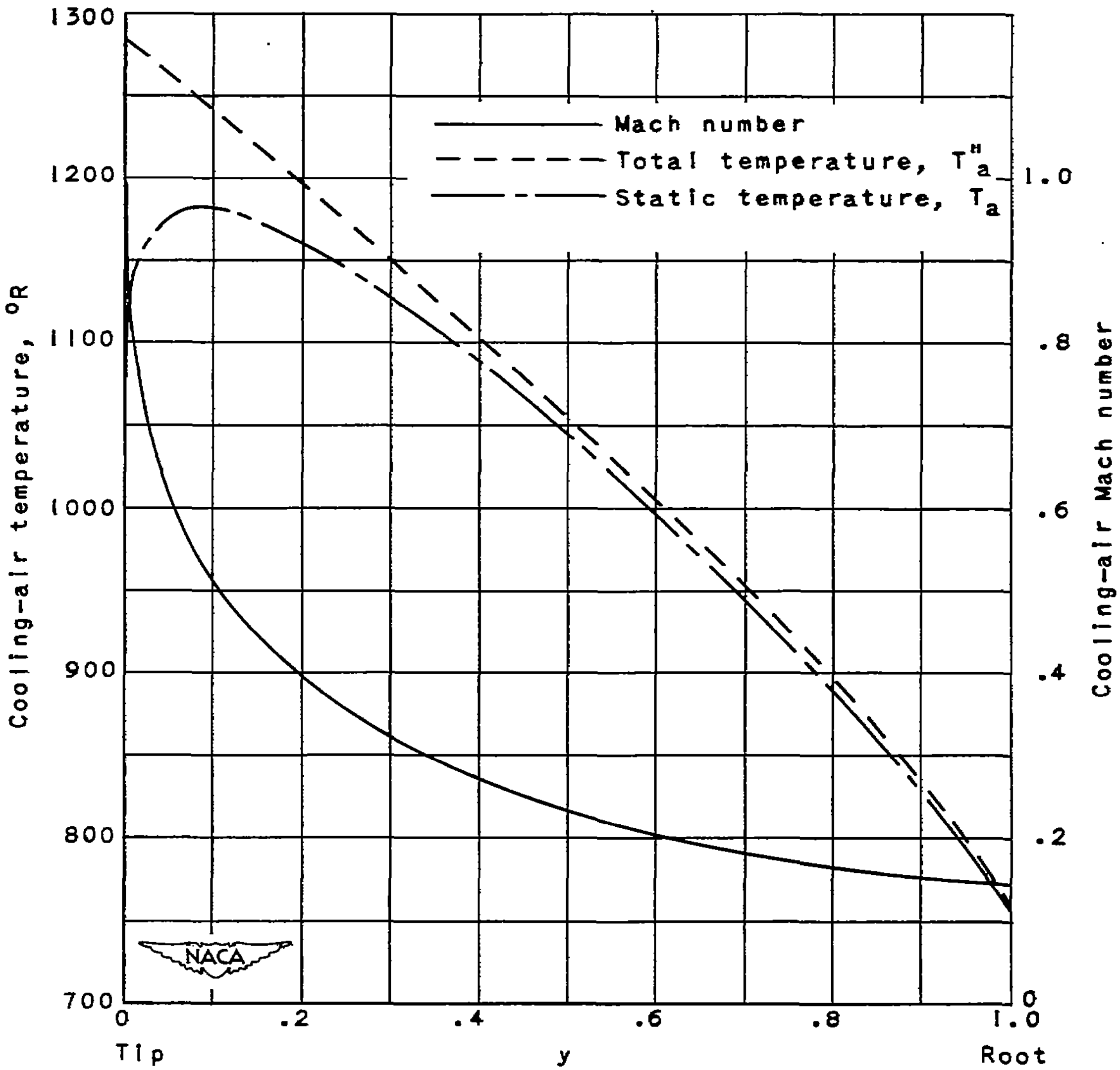
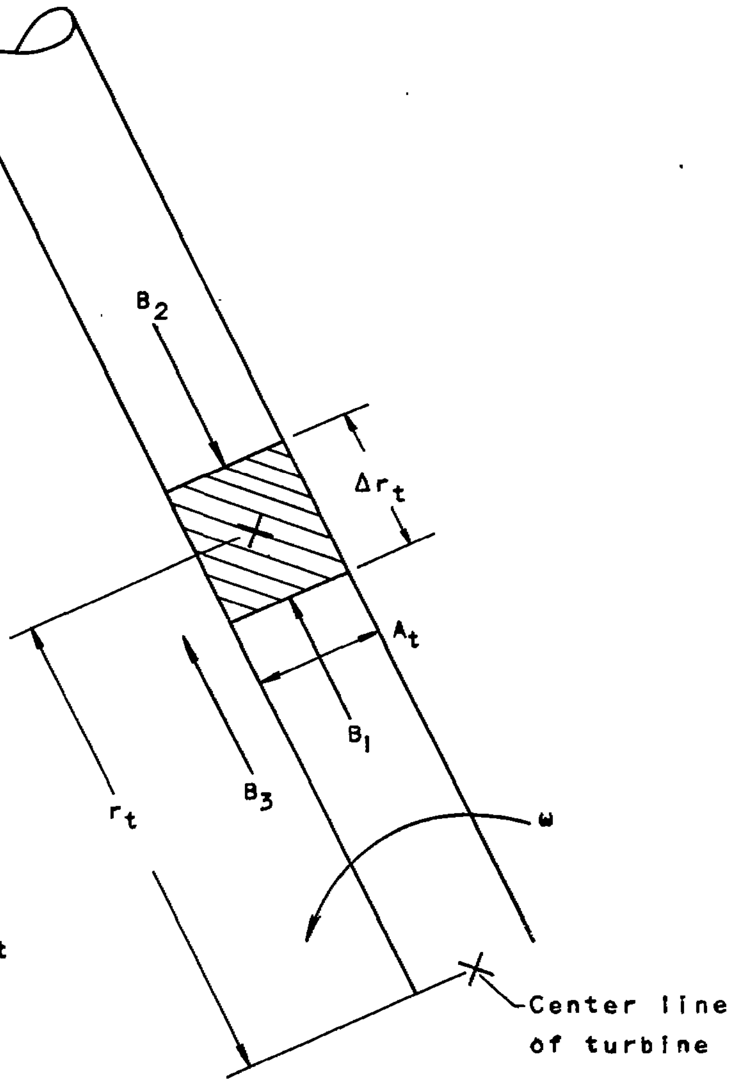



Figure 6. - Variation of Mach number, total temperature, and static temperature of cooling air from blade tip to root.




$$B_1 = p_a A_t$$

$$B_2 = \left(p_a + \frac{dp_a}{dr_t}\Delta r_t\right) A_t$$

$$B_3 = (\rho_{a,t} A_t \Delta r_t)\, \omega^2 r_t$$

Figure 7. - Diagrammatic sketch showing forces in rotating pressure tube.